

ELECTROSTATICS-01

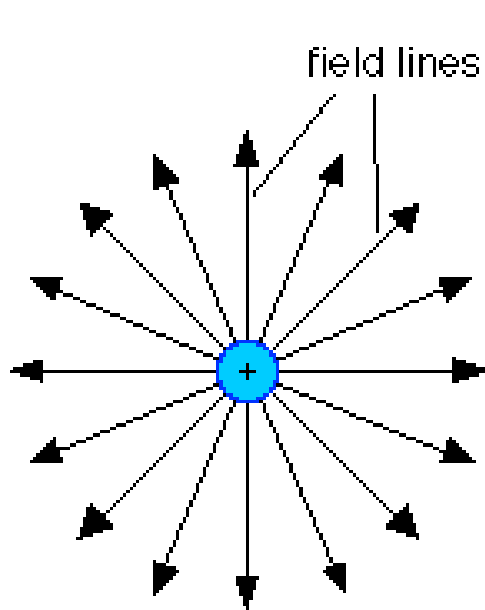
Electric field lines

Integral and differential form of
Gauss Law

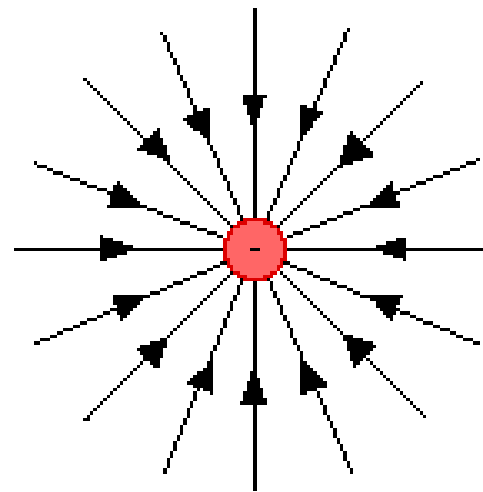
ELECTRIC FIELD LINES

$$E = k \frac{q}{r^2} \hat{r}$$

- Electric field varies Inversely with distance squared.
- The magnitude of field is determined by the density of field lines.



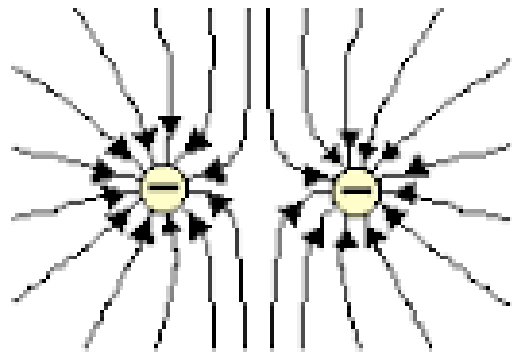
The electric field from an isolated positive charge



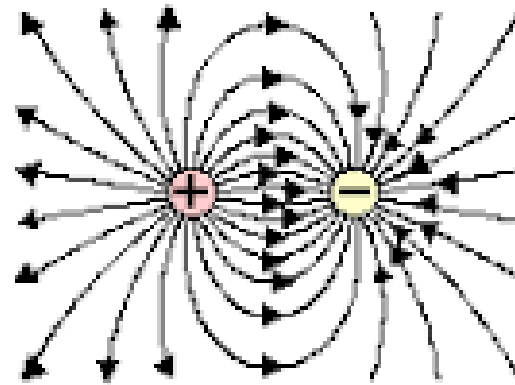
The electric field from an isolated negative charge

$$\text{Density of lines} = \frac{n}{4\pi r^2}$$

Other Charge Configurations

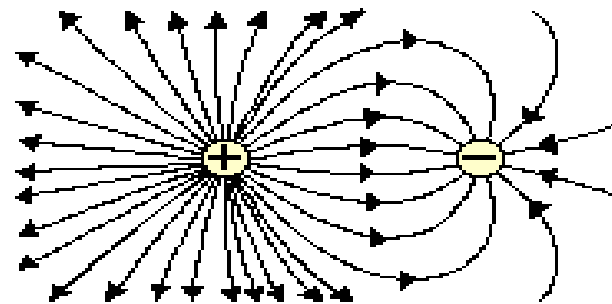
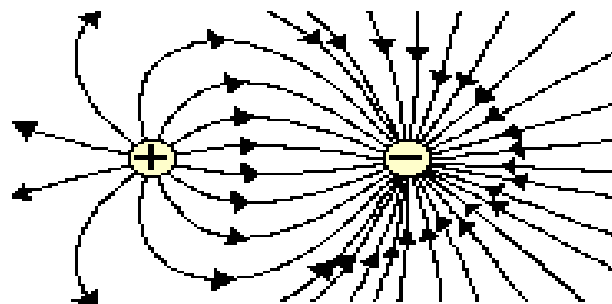
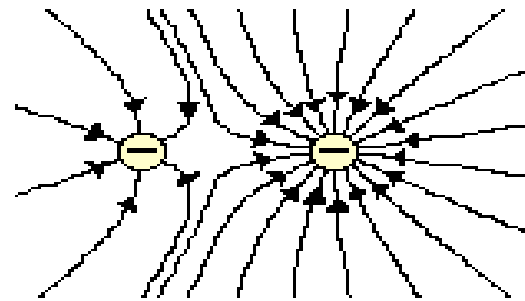
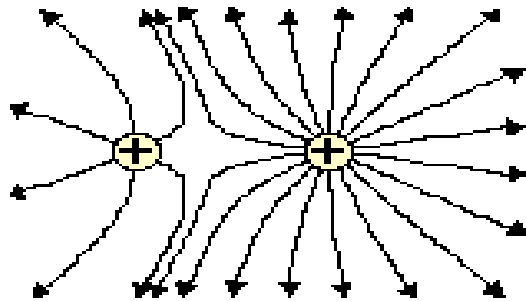


Two Negatively-Charged Objects



A Positively and a Negatively-Charged Object

Electric Field Line Patterns for Objects with Unequal Amounts of Charge



Properties of Lines of Force

- Field lines starts from positive charge and end at negative charge.
- Tangent of field line represent direction of field at particular point.
- Continuous curves if no conductor is present.
- Lines of force do not cross each other.

Differential equation for Field lines

Tangent represent direction E field

$$\frac{d\vec{r}}{dS} = a\vec{E}(r)$$

$$\frac{dx}{dS} = aE_x \quad \frac{dy}{dS} = aE_y \quad \frac{dz}{dS} = aE_z$$

$$\frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z}$$

FLUX AND GAUSS LAW

- Flux is a measure of number of field lines passing through surface S.
- It also measure total charge enclosed inside a closed surface.

$$\Phi_E = \oint E \cdot da$$

- For a isolated charge,

$$\begin{aligned}\oint E \cdot da &= \int \frac{1}{4\pi \epsilon_0} \left(\frac{q}{r^2} \hat{r} \right) \cdot (r^2 \sin\theta d\theta d\phi \hat{r}) \\ &= \frac{1}{\epsilon_0} q\end{aligned}$$

- For bunch of scattered charges,

$$E = \sum_{i=1}^n E_i$$

$$\oint E \cdot da = \oint \sum_{i=1}^n E_i \cdot da = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i$$

$$\oint E \cdot da = \frac{1}{\epsilon_0} q_{\text{enclosed}}$$

- This is integral form of Gauss law

$$\oint E \cdot da = \iiint (\nabla \cdot E) d\tau \quad q = \iiint \rho d\tau$$

$$\iiint (\nabla \cdot E) d\tau = \frac{1}{\epsilon_0} \iiint \rho d\tau$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

- This is differential form of Gauss Law